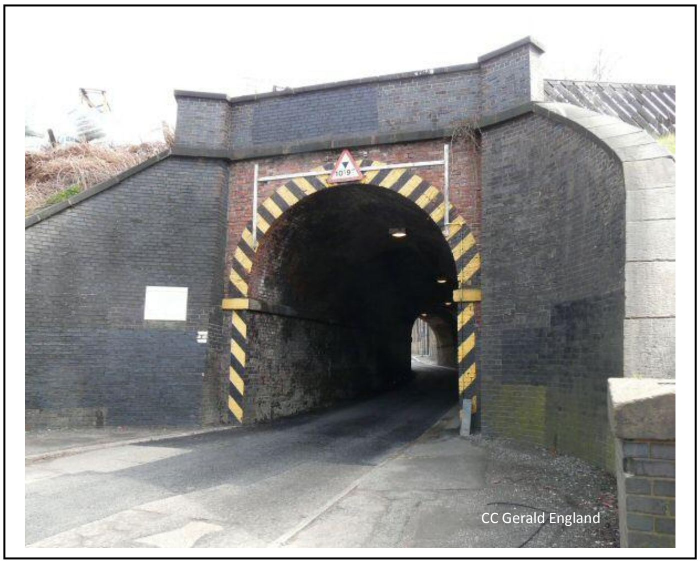
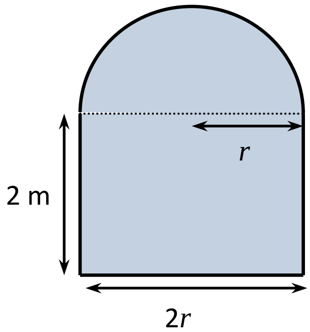
This activity is about using a graphical or algebraic method to solve problems in real contexts that can be modelled using quadratic expressions.

The first problem is about a road tunnel. The information sheet shows how you could solve the problem.



Road tunnel information sheet

A road tunnel is designed to have a cross-section consisting of a rectangle of height 2 metres below a semi-circle as shown.



The area of the cross-section must be at least 16 m^2 to allow adequate ventilation.

The strength of the materials used to support the tunnel suggests that the area must be no more than 32 m^2 because of the danger of collapse.

Think about

From the wording of the information, do you think that area values of exactly 16 m^2 and exactly 32 m^2 are acceptable values?

Before you read any further, see if you can work out how to solve the problem on your own, and try to do so. If you get stuck, read on.

Using the information

You will need to use the values given for the area A in order to find the radius r.

First you need a formula for the area in terms of r.

The area of the semi-circle is $0.5\pi r^2$.

The area of the rectangle is $2 \times 2r = 4r$.

So the formula for the area of the tunnel entrance is $A = 0.5\pi r^2 + 4r$

The problem can now be solved either by drawing a graph or by using algebra.

A Graphical method

This method uses a graph to find the minimum and maximum values of the radius, r metres, and then the range of possible road widths.
The graph can be drawn in Excel or on graph paper.

Think about

What spreadsheet formulae can you use to draw a graph of A against r for values of r up to 4 metres?

Try this

1 Put headings r (m) and A (m^2) at the top of columns A and B.

Use 0 as the first value of r. Enter 0 in cell A2.

Write a spreadsheet formula in cell B2 to calculate the area.

See formulae below.

Write a spreadsheet formula in cell A3 to calculate the next value of r.

	A	B	C
1	r (m)	A(m^2)	
2	0	0	
3	0.5		

Numerical values

	A	B	C
1	r (m)	A(m^2)	
2	0	=0.5*PI()*A2^2+4*A2	
3	=A2+0.5		

Formulae

2 Use ***'fill down'*** to complete the table as far as $r = 4$.
The results are shown below, with the r column formatted to show 1 decimal place and the A column to show 2 decimal places:

	A	B	C
1	r (m)	A(m^2)	
2	0.0	0.00	
3	0.5	2.39	
4	1.0	5.57	
5	1.5	9.53	
6	2.0	14.28	
7	2.5	19.28	
8	3.0	26.14	
9	3.5	33.24	
10	4.0	41.13	

Numerical values

	A	B	C
1	r (m)	A(m^2)	
2	0.0	=0.5*PI()*A2^2+4*A2	
3	=A2+0.5	=0.5*PI()*A3^2+4*A3	
4	=A3+0.5	=0.5*PI()*A4^2+4*A4	
5	=A4+0.5	=0.5*PI()*A5^2+4*A5	
6	=A5+0.5	=0.5*PI()*A6^2+4*A6	
7	=A6+0.5	=0.5*PI()*A7^2+4*A7	
8	=A7+0.5	=0.5*PI()*A8^2+4*A8	
9	=A8+0.5	=0.5*PI()*A9^2+4*A9	
10	=A9+0.5	=0.5*PI()*A10^2+4*A10	

Formulae

3 Use the values in columns A and B to draw a graph (select the curve with points in the Scatter option). Add a title and labels.

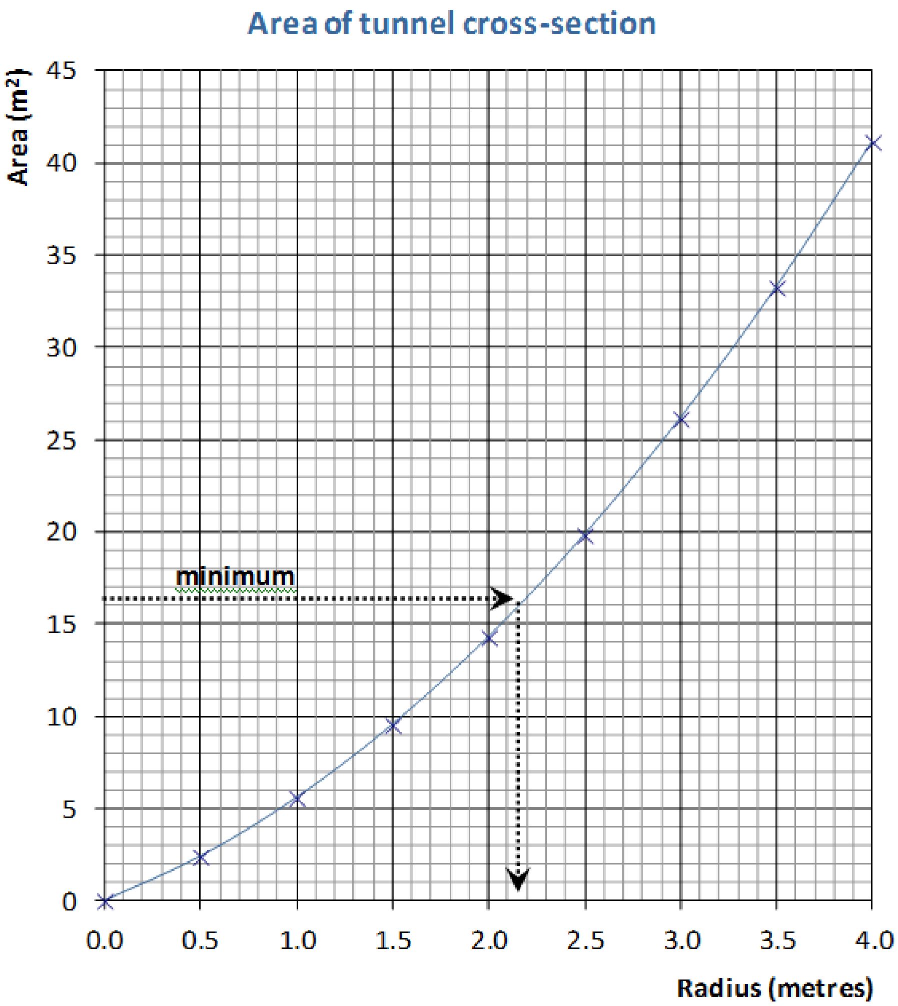
4 Include both major and minor gridlines. Format them so that it is easy to read off values of A and r.

> **Think about ...**
> how to use the graph to find the maximum and minimum values of r.

5a The graph can be used to find the minimum value of r, using the minimum value of 16 m^2 for the area of the tunnel entrance.

The minimum value of r is about 2.16 metres as shown below. Check this on your graph.

The road width is $2r$. What is the minimum road width?



5b The maximum area is 32 m^2.

Use the graph to find the maximum value of r, then the maximum road width.

Draw lines on your graph to show the maximum and minimum values of r.
You can print your graph then do this by hand, or use the drawing tools in Excel before printing the graph.

B Algebraic method

The formula for the area of the tunnel entrance is $A = 0.5\,\pi r^2 + 4r$

First the radius that will give an area of 16 m^2 can be found as shown below:

For an area of 16 m^2 $\quad 0.5\,\pi r^2 + 4r = 16$

This rearranges to $\quad 0.5\,\pi r^2 + 4r - 16 = 0$

In the quadratic formula: $a = 0.5\pi$, $b = 4$, $c = -16$

Using the formula:

$$r = \frac{-4 \pm \sqrt{4^2 - 4 \times 0.5 \times \pi \times -16}}{2 \times 0.5 \times \pi}$$

$$r = \frac{-4 \pm \sqrt{116.53}}{\pi} = \frac{-4 \pm 10.795}{\pi}$$

Quadratic formula

Solutions of $ax^2 + bx + c = 0$ are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The radius must be positive so $\quad r = \dfrac{-4 + 10.795}{\pi} = \dfrac{6.795}{\pi} = 2.163$

This gives a minimum value for the radius of 2.163 metres.

The minimum tunnel width = $2r = 2 \times 2.163 = 4.33$

The minimum width of the road tunnel is 4.3 metres (to 1 decimal place).

Try this

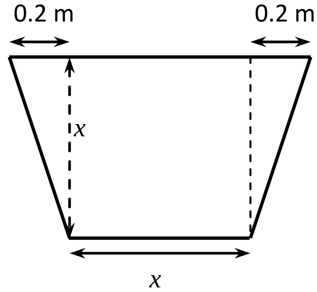
Use a similar method to find the value of r that will give an area of 32 m^2.
Use this value of r to find the maximum width of the tunnel.

Try these

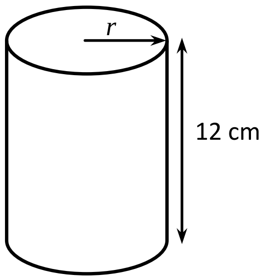
1 The sketch shows the cross-section of a design for a waste skip.

a Show that the area of the cross-section is given by the formula $A = x^2 + 0.2x$, where x is the length of the base and height in metres.

b In order that the skip should have the required volume, the cross-sectional area must be 2.5 square metres. Find the value of x.



2 A container is to be in the shape of a cylinder of height 12 cm.

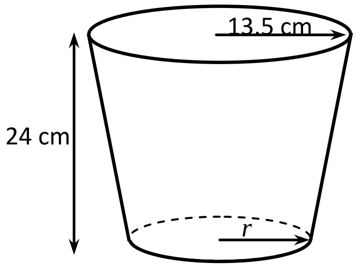


a Explain why the total surface area of the container is given by:

$A = 2\pi r^2 + 24\pi r$

b The manufacturer wants to limit the surface area of the container to 300 cm^2. Find the maximum radius.

3 The formula for the volume of a bucket of height h is

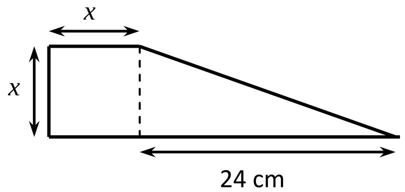


$V = \frac{1}{3}\pi h\left(R^2 + Rr + r^2\right)$ where R and r are the radii of the ends.

a A bucket is designed to be 24 cm high and to have a top with radius 13.5 cm. Show that for this bucket: $V = 8\pi\left(182.25 + 13.5r + r^2\right)$

b The bucket is required to have a volume of 10 litres, where 1 litre = 1000 cm^3. Find the radius of the bottom of the bucket.

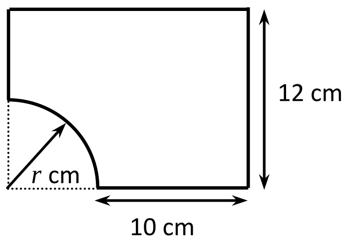
4 The sketch shows the cross-section of a wedge.



Find the value of x that would give a cross-sectional area of:

a 150 cm^2 **b** 275 cm^2

5 A quarter circle is to be removed from a rectangular metal plate to give the shape shown in the sketch.



a Show that the remaining area is $A = 120 + 12r - 0.25\pi r^2$

b It is required that the area should be 140 cm^2. Find the value of r.

At the end of the activity

- Describe the steps in the graphical method.
- Describe the steps in the algebraic method.
- Which of the methods do you think is easier? Why?
  Was it the same for every question?
- If you found any part of the questions difficult,
  what could you do to improve your mathematical skills?